Feuille de calcul n°8 — Trigonométrie — Corrigé

Exercice 1. Donner les valeurs de :

$$A=\cos\left(\frac{3\pi}{4}\right) \qquad B=\tan\left(\frac{\pi}{4}\right) \qquad C=\tan\left(\frac{5\pi}{4}\right)$$

$$D=\cos(7\pi) \qquad E=\cos\left(\frac{\pi}{6}\right) \qquad F=\sin\left(\frac{7\pi}{6}\right).$$

Solution.

$A=\cos\left(\pi-\frac{\pi}{4}\right)=-\cos\left(\frac{\pi}{4}\right)$ donc $\boxed{A=-\frac{\sqrt{2}}{2}}$.

$\boxed{B=1}$.

$C=\tan\left(\pi+\frac{\pi}{4}\right)=\tan\left(\frac{\pi}{4}\right)$ donc $\boxed{C=1}$.

$D=\cos(6\pi+\pi)=\cos(\pi)$ donc $\boxed{D=-1}$.

$\boxed{E=\frac{\sqrt{3}}{2}}$.

$F=\sin\left(\pi+\frac{\pi}{6}\right)=-\sin\left(\frac{\pi}{6}\right)$ donc $\boxed{F=-\frac{1}{2}}$.

Exercice 2. Calculer :

$$A=\sin\left(\frac{5\pi}{6}\right)+\sin\left(\frac{7\pi}{6}\right) \qquad B=\cos^2\left(\frac{4\pi}{3}\right)+\sin^2\left(\frac{4\pi}{3}\right) \qquad C=\cos^2\left(\frac{4\pi}{3}\right)-\sin^2\left(\frac{4\pi}{3}\right)$$

$$D=\cos\left(\frac{\pi}{4}\right)+\cos\left(\frac{3\pi}{4}\right)+\cos\left(\frac{5\pi}{4}\right)+\cos\left(\frac{7\pi}{4}\right) \qquad E=\tan\left(\frac{2\pi}{3}\right)+\tan\left(\frac{3\pi}{4}\right)+\tan\left(\frac{5\pi}{6}\right)+\tan\left(\frac{7\pi}{6}\right)$$

Solution.

$A=\sin\left(\pi-\frac{\pi}{6}\right)+\sin\left(\pi+\frac{\pi}{6}\right)=\sin\left(\frac{\pi}{6}\right)-\sin\left(\frac{\pi}{6}\right)$ donc $\boxed{A=0}$.

$\boxed{B=1}$ (car, pour tout réel x, $\cos^2(x)+\sin^2(x)=1$).

$C=\cos\left(2\times\frac{4\pi}{3}\right)=\cos\left(\frac{8\pi}{3}\right)=\cos\left(2\pi+\frac{2\pi}{3}\right)=\cos\left(\frac{2\pi}{3}\right)=\cos\left(\pi-\frac{\pi}{3}\right)=-\cos\left(\frac{\pi}{3}\right)$ donc $\boxed{C=-\frac{1}{2}}$.

$$\begin{aligned}D&=\cos\left(\frac{\pi}{4}\right)+\cos\left(\pi-\frac{\pi}{4}\right)+\cos\left(\pi+\frac{\pi}{4}\right)+\cos\left(2\pi-\frac{\pi}{4}\right)\\&=\cos\left(\frac{\pi}{4}\right)-\cos\left(\frac{\pi}{4}\right)-\cos\left(\frac{\pi}{4}\right)+\cos\left(-\frac{\pi}{4}\right)=-\cos\left(\frac{\pi}{4}\right)+\cos\left(\frac{\pi}{4}\right)\end{aligned}$$

donc $\boxed{D=0}$.

$$\begin{aligned}E &= \tan\left(\pi-\frac{\pi}{3}\right)+\tan\left(\pi-\frac{\pi}{4}\right)+\tan\left(\pi-\frac{\pi}{6}\right)+\tan\left(\pi+\frac{\pi}{6}\right)\\ &= \tan\left(-\frac{\pi}{3}\right)+\tan\left(-\frac{\pi}{4}\right)+\tan\left(-\frac{\pi}{6}\right)+\tan\left(\frac{\pi}{6}\right)\\ &= -\tan\left(\frac{\pi}{3}\right)-\tan\left(\frac{\pi}{4}\right)-\tan\left(\frac{\pi}{6}\right)+\tan\left(\frac{\pi}{6}\right)\end{aligned}$$

donc $\boxed{E=-\sqrt{3}-1}$.

Exercice 3. Soit $x \in \mathbb{R}$. Simplifier :

$$A=\sin(\pi-x)+\cos\left(\frac{\pi}{2}+x\right) \qquad B=\sin(-x)+\cos(\pi+x)+\sin\left(\frac{\pi}{2}-x\right)$$
$$C=\sin\left(\frac{\pi}{2}-x\right)+\sin\left(\frac{\pi}{2}+x\right) \qquad D=\cos(x-\pi)+\sin\left(-\frac{\pi}{2}-x\right)$$

Solution.

$A=\sin(x)-\sin(x)$ donc $\boxed{A=0}$.

$B=-\sin(x)-\cos(x)+\cos(x)$ donc $\boxed{B=-\sin(x)}$.

$C=\cos(x)+\cos(x)$ donc $\boxed{C=2\cos(x)}$

$D=-\cos(x)-\cos(x)$ donc $\boxed{D=-2\cos(x)}$.

Exercice 4. Sans chercher à les calculer, déterminer le signe de chacun des nombres suivants.

$$A=\cos\left(\frac{2\pi}{5}\right) \qquad B=\sin\left(\frac{8\pi}{5}\right) \qquad C=\sin\left(\frac{14\pi}{5}\right)$$
$$D=\sin\left(\frac{7\pi}{5}\right) \qquad E=\tan\left(\frac{13\pi}{5}\right) \qquad F=\tan\left(-\frac{3\pi}{5}\right)$$

Solution.

Comme $-\frac{\pi}{2}<\frac{2\pi}{5}<\frac{\pi}{2}$, $\boxed{A>0}$.

Comme $\pi<\frac{8\pi}{5}<2\pi$, $\boxed{B<0}$.

$C=\sin\left(2\pi+\frac{4\pi}{5}\right)=\sin\left(\frac{4\pi}{5}\right)$ donc, comme $0<\frac{4\pi}{5}<\pi$, $\boxed{C>0}$.

Comme $\pi<\frac{7\pi}{5}<2\pi$, $\boxed{D<0}$.

$E=\tan\left(\pi+\frac{3\pi}{5}\right)=\tan\left(\frac{3\pi}{5}\right)$. Or, comme $\frac{\pi}{2}<\frac{3\pi}{5}<\pi$, $\cos\left(\frac{3\pi}{5}\right)<0$ et $\sin\left(\frac{3\pi}{5}\right)>0$ donc $\boxed{E<0}$.

$F=-\tan\left(\frac{3\pi}{5}\right)=-E$ donc $\boxed{F>0}$.

Exercice 5. Soit $x \in \left]0\,;\frac{\pi}{2}\right[$. En utilisant les formules de duplication, simplifier :

$$A=\frac{1-\cos(2x)}{\sin(2x)} \qquad B=\frac{\cos(2x)}{\cos(x)}-\frac{\sin(2x)}{\sin(x)}$$

Solution.
$A = \dfrac{1-(1-2\sin^2(x))}{2\sin(x)\cos(x)} = \dfrac{2\sin^2(x)}{2\sin(x)\cos(x)} = \dfrac{\sin(x)}{\cos(x)}$ donc $\boxed{A = \tan(x)}$.
$B = \dfrac{\sin(x)\cos(2x)-\sin(2x)\cos(x)}{\cos(x)\sin(x)} = \dfrac{\sin(x-2x)}{\sin(x)\cos(x)} = \dfrac{-\sin(x)}{\cos(x)\sin(x)}$ donc $\boxed{B = -\dfrac{1}{\cos(x)}}$.

Exercice 6. Résoudre dans $\mathbb{R}$ les équations suivantes d'inconnues x.

$$(E_1) : \cos(x) = \frac{1}{2} \qquad (E_2) : \sin(x) = -\frac{\sqrt{3}}{2} \qquad (E_3) : \sin(x) = \cos\left(\frac{2\pi}{3}\right) \qquad \cos^2(x) = \frac{1}{2}.$$

Solution.
$(E_1) \iff \cos(x) = \cos\left(\frac{\pi}{3}\right) \iff x = \frac{\pi}{3}\ [2\pi]$ ou $x = -\frac{\pi}{3}\ [2\pi]$
Ainsi, $\boxed{\text{l'ensemble des solutions de } (E_1) \text{ est } \left\{\frac{\pi}{3} + k2\pi \mid k \in \mathbb{Z}\right\} \cup \left\{-\frac{\pi}{3} + k2\pi \mid k \in \mathbb{Z}\right\}}$.

$(E_2) \iff \sin(x) = \sin\left(-\frac{\pi}{3}\right) \iff x = -\frac{\pi}{3}\ [2\pi]$ ou $x = -\frac{2\pi}{3}\ [2\pi]$
Ainsi, $\boxed{\text{l'ensemble des solutions de } (E_2) \text{ est } \left\{-\frac{\pi}{3} + k2\pi \mid k \in \mathbb{Z}\right\} \cup \left\{-\frac{2\pi}{3} + k2\pi \mid k \in \mathbb{Z}\right\}}$.

$(E_3) \iff \sin(x) = \sin\left(-\frac{\pi}{6}\right) \iff x = -\frac{\pi}{6}\ [2\pi]$ ou $x = -\frac{5\pi}{6}\ [2\pi]$
Ainsi, $\boxed{\text{l'ensemble des solutions de } (E_3) \text{ est } \left\{-\frac{\pi}{6} + k2\pi \mid k \in \mathbb{Z}\right\} \cup \left\{-\frac{5\pi}{6} + k2\pi \mid k \in \mathbb{Z}\right\}}$.

$(E_4) \iff |\cos(x)| = \frac{1}{\sqrt{2}} \iff \cos(x) = \frac{\sqrt{2}}{2}$ ou $= \cos(x) = \frac{\sqrt{2}}{2}$
$\iff x = \frac{\pi}{4}\ [2\pi]$ ou $x = -\frac{\pi}{4}\ [2\pi]$ pu $x = \frac{3\pi}{4}\ [2\pi]$ ou $x = -\frac{3\pi}{4}\ [2\pi]$
$\iff x = \frac{\pi}{4}\ \left[\frac{\pi}{2}\right]$
Ainsi, $\boxed{\text{l'ensemble des solutions de } (E_4) \text{ est } \left\{\frac{\pi}{4} + k\frac{\pi}{2} \mid k \in \mathbb{Z}\right\}}$.