

## Feuille de calcul n°3 — Calcul littéral

**Exercice 1.** Développer, réduire et ordonner chacune des expressions suivantes.

$$A = (x^2 - x)(x + 1)$$

$$B = (2x^2 + x - 4)(x + 2)$$

$$C = (2x^2 + 3 - 4x)(2x + 4)$$

$$D = (x + 1)(3 - 2x)(x^2 - 2)$$

**Solution.**

$$A = x^3 + x^2 - x^2 - x \text{ donc } \boxed{A = x^3 - x}.$$

$$B = 2x^3 + 4x^2 + x^2 + 2x - 4x - 8 \text{ donc } \boxed{B = 2x^3 + 5x^2 - 2x - 8}.$$

$$C = 4x^3 + 8x^2 + 6x + 12 - 8x^2 - 16x \text{ donc } \boxed{C = 4x^3 - 10x + 12}.$$

$$D = (3x - 2x^2 + 3 - 2x)(x^2 - 2) = (-2x^2 + x + 3)(x^2 - 2) = -2x^4 + 4x^2 + x^3 - 2x + 3x^2 - 6$$

donc  $\boxed{D = -2x^4 + x^3 + 7x^2 - 2x - 6}$ .

**Exercice 2.** Développer, réduire et ordonner chacune des expressions suivantes.

$$A = (6 - 3x)^2$$

$$B = (1 + 8x)^2$$

$$C = (4x + 5)(5 - 4x)$$

$$D = (7 - 4x)^2$$

$$E = (-2x - 9)^2$$

$$F = (6 - 2x)(6 + 2x)$$

$$G = \left(3x - \frac{4}{3}\right)^2$$

$$H = (5x - 3)^2 - (3x - 7)^2$$

**Solution.**

$$A = 6^2 - 2 \times 6 \times 3x + (3x)^2 \text{ donc } \boxed{A = 9x^2 - 36x + 36}.$$

$$B = 1^2 + 2 \times 1 \times 8x + (8x)^2 \text{ donc } \boxed{B = 64x^2 + 16x + 1}.$$

$$C = (5 + 4x)(5 - 4x) = 5^2 - (4x)^2 \text{ donc } \boxed{C = -16x^2 + 25}.$$

$$D = 7^2 - 2 \times 7 \times 4x + (4x)^2 \text{ donc } \boxed{D = 16x^2 - 56x + 49}.$$

$$E = (-2x)^2 - 2 \times (-2x) \times 9 + 9^2 \text{ donc } \boxed{E = 4x^2 + 36x + 81}.$$

$$F = 6^2 - (2x)^2 \text{ donc } \boxed{F = -4x^2 + 36}.$$

$$G = (3x)^2 - 2 \times 3x \times \frac{4}{3} + \left(\frac{4}{3}\right)^2 \text{ donc } \boxed{G = 9x^2 - 8x + \frac{16}{9}}.$$

$$H = (5x)^2 - 2 \times 5x \times 3 + 3^2 - ((3x)^2 - 2 \times 3x \times 7 + 7^2) = 25x^2 - 30x + 9 - (9x^2 - 42x + 49)$$

donc  $\boxed{H = 16x^2 + 12x - 40}$ .

**Exercice 3.** Factoriser les expressions suivantes.

$$A = 8a^2 - 24a + 32a^3$$

$$B = 3a^2x - 6ax^2 + 12abx$$

$$C = 5a^4b^3 + 2a^2x^3 - 3a^2b^5$$

$$D = (2x - 3)(5x - 1) - (2x - 3)(x + 1)$$

$$E = a^2 - 25$$

$$F = 4x^2 - 1$$

**Solution.**

$$A = 8a(a - 3 + 4a^2) \text{ donc } \boxed{A = 8a(4a^2 + a - 3)}.$$

$$\boxed{B = 3ax(a - 2x + 4b)}$$

$$C = a^2(5a^2b^3 + 2x^3 - 3b^5)$$

$$D = (2x - 3)[(5x - 1) - (x + 1)] = (2x - 3)(5x - 1 - x - 1) = (2x - 3)(4x - 2) \text{ donc}$$

$$D = 2(2x - 3)(2x - 1).$$

$$E = a^2 - 5^2 \text{ donc } E = (a - 5)(a + 5).$$

$$F = (2x)^2 - 1^2 \text{ donc } F = (2x - 1)(2x + 1).$$

**Exercice 4.** Factoriser les expressions suivantes.

$$A = (7x - 1)^2 - (7x - 1)(3x + 2)$$

$$B = (4 - 3x)(2 + 3x) - 2(1 - 2x)(3x - 4)$$

$$C = (x - 8)(4x - 1) + x^2 - 8x$$

$$D = x^2 - x + (x + 1)(1 - x)$$

$$E = 25a^2 - 16b^2$$

$$F = 49x^2 - 25$$

**Solution.**

$$A = (7x - 1)(7x - 1) - (7x - 1)(3x + 2) = (7x - 1)[(7x - 1) - (3x + 2)] = (7x - 1)(7x - 1 - 3x - 2)$$

donc  $A = (7x - 1)(4x - 3).$

$$B = (4 - 3x)(2 + 3x) - 2(1 - 2x)[-(4 - 3x)] = (4 - 3x)(2 + 3x) + 2(1 - 2x)(4 - 3x)$$
$$= (4 - 3x)[(2 + 3x) + 2(1 - 2x)] = (4 - 3x)(2 + 3x + 2 - 4x) \text{ donc } B = (4 - 3x)(-x + 4).$$

$$C = (x - 8)(4x - 1) + x(x - 8) = (x - 8)[(4x - 1) + x] \text{ donc } C = (x - 8)(5x - 1).$$

$$D = x(x - 1) + (x + 1)[-(x - 1)] = x(x - 1) - (x + 1)(x - 1) = (x - 1)[x - (x + 1)]$$
$$= (x - 1)(x - x - 1) \text{ donc } D = 1 - x.$$

$$E = (5a)^2 - (4b)^2 \text{ donc } E = (5a - 4b)(5a + 4b).$$

$$F = (7a)^2 - 5^2 \text{ donc } F = (7a - 5)(7a + 5).$$

**Exercice 5.** Factoriser les expressions suivantes.

$$A = (x - 2)^4 - (x - 2)^3$$

$$B = 2(x - 3)^3 + (2x - 6)(7 - 3x)^2$$

$$C = (3x + 1)(2x - 3) + (3x + 1)(x + 2) - (5x + 4)(3x + 1)$$

$$D = (49x^2 - 1)(x + 1)(3 - 2x) + (2 - 14x)(x^2 - 1)$$

**Solution.**

$$A = (x - 2)^3 \times (x - 2) - (x - 2)^3 \times 1 = (x - 2)^3 [(x - 2) - 1] \text{ donc } A = (x - 2)^3(x - 3).$$

$$B = 2(x - 3)(x - 3)^2 + 2(x - 3)(7 - 3x)^2 = 2(x - 3)[(x - 3)^2 + (7 - 3x)^2]$$
$$= 2(x - 3)[x^2 - 6x + 9 + (49 - 42x + 9x^2)] = 2(x - 3)(10x^2 - 48x + 58)$$

donc  $B = 4(x - 3)(5x^2 - 24x + 29).$

$$C = (3x + 1)[(2x - 3) + (x + 2) - (5x + 4)] \text{ donc } C = (3x + 1)(-2x - 5).$$

$$D = ((7x)^2 - 1^2)(x + 1)(3 - 2x) + 2(1 - 7x)(x^2 - 1^2)$$
$$= (7x - 1)(7x + 1)(x + 1)(3 - 2x) - 2(7x - 1)(x - 1)(x + 1)$$
$$= (7x - 1)(x + 1)[(7x + 1)(3 - 2x) - 2(x - 1)]$$
$$= (7x - 1)(x + 1)[21x - 14x^2 + 3 - 2x - 2x + 2]$$

donc  $D = (7x - 1)(x + 1)(-14x^2 + 17x + 5).$

**Exercice 6.** Effectuer les calculs suivants en simplifiant au maximum le résultat.

$$A = \frac{x + 2}{x + 5} - \frac{1 - x}{x - 2}$$

$$B = \frac{x^2 + 1}{10 - 2x} + \frac{5 + x}{5 - x}$$

$$C = \frac{x + 1}{x - 1} \times \left(1 - \frac{2}{x + 1}\right)$$

$$D = \frac{\frac{x+1}{x-1} - \frac{2x+1}{2x-1}}{\frac{2x+1}{x-1} - \frac{x+1}{2x-1}}.$$

**Solution.**

$$A = \frac{(x+2)(x-2)}{(x+5)(x-2)} - \frac{(1-x)(x+5)}{(x-2)(x+5)} = \frac{x^2 - 4 - (x+5 - x^2 - 5x)}{(x-2)(x+5)} = \frac{x^2 - 4 + x^2 + 4x - 5}{(x-2)(x+5)}$$

donc  $A = \frac{2x^2 + 4x - 9}{(x-2)(x+5)}$ .

$$B = \frac{x^2 + 1}{2(5-x)} + \frac{5+x}{5-x} = \frac{x^2 + 1 + 2(5+x)}{2(5-x)} \quad \text{donc } B = \frac{x^2 + 2x + 11}{2(5-x)}.$$

$$C = \frac{x+1}{x-1} \times \left( \frac{x+1}{x+1} - \frac{2}{x+1} \right) = \frac{x+1}{x-1} \times \frac{x+1-2}{x+1} = \frac{x+1}{x-1} \times \frac{x-1}{x+1} \quad \text{donc } C = 1.$$

$$D = \frac{\frac{(x+1)(2x-1) - (2x+1)(x-1)}{(x-1)(2x-1)}}{\frac{(2x+1)(2x-1) - (x+1)(x-1)}{(x-1)(2x-1)}} = \frac{(x+1)(2x-1) - (2x+1)(x-1)}{(2x+1)(2x-1) - (x+1)(x-1)}$$
$$= \frac{2x^2 - x + 2x - 1 - (2x^2 - 2x + x - 1)}{(2x)^2 - 1^2 - (x^2 - 1^2)} = \frac{2x^2 + x - 1 - 2x^2 + x + 1}{4x^2 - 1 - x^2 + 1} = \frac{2x}{3x^2}$$

donc  $D = \frac{2}{3x}$