

Feuille de calcul n°10 — Trigonométrie — Corrigé

Exercice 1. Donner les valeurs de :

$$\begin{aligned} A &= \cos\left(\frac{3\pi}{4}\right) & B &= \tan\left(\frac{\pi}{4}\right) & C &= \tan\left(\frac{5\pi}{4}\right) \\ D &= \cos(7\pi) & E &= \cos\left(\frac{\pi}{6}\right) & F &= \sin\left(\frac{7\pi}{6}\right). \end{aligned}$$

Solution.

$$A = \cos\left(\pi - \frac{\pi}{4}\right) = -\cos\left(\frac{\pi}{4}\right) \text{ donc } \boxed{A = -\frac{\sqrt{2}}{2}}.$$

$$\boxed{B = 1}.$$

$$C = \tan\left(\pi + \frac{\pi}{4}\right) = \tan\left(\frac{\pi}{4}\right) \text{ donc } \boxed{C = 1}.$$

$$D = \cos(6\pi + \pi) = \cos(\pi) \text{ donc } \boxed{D = -1}.$$

$$\boxed{E = \frac{\sqrt{3}}{2}}.$$

$$F = \sin\left(\pi + \frac{\pi}{6}\right) = -\sin\left(\frac{\pi}{6}\right) \text{ donc } \boxed{F = -\frac{1}{2}}.$$

Exercice 2. Calculer :

$$A = \sin\left(\frac{5\pi}{6}\right) + \sin\left(\frac{7\pi}{6}\right) \quad B = \cos^2\left(\frac{4\pi}{3}\right) + \sin^2\left(\frac{4\pi}{3}\right) \quad C = \cos^2\left(\frac{4\pi}{3}\right) - \sin^2\left(\frac{4\pi}{3}\right)$$

$$D = \cos\left(\frac{\pi}{4}\right) + \cos\left(\frac{3\pi}{4}\right) + \cos\left(\frac{5\pi}{4}\right) + \cos\left(\frac{7\pi}{4}\right) \quad E = \tan\left(\frac{2\pi}{3}\right) + \tan\left(\frac{3\pi}{4}\right) + \tan\left(\frac{5\pi}{6}\right) + \tan\left(\frac{7\pi}{6}\right)$$

Solution.

$$A = \sin\left(\pi - \frac{\pi}{6}\right) + \sin\left(\pi + \frac{\pi}{6}\right) = \sin\left(\frac{\pi}{6}\right) - \sin\left(\frac{\pi}{6}\right) \text{ donc } \boxed{A = 0}.$$

$$\boxed{B = 1} \text{ (car, pour tout réel } x, \cos^2(x) + \sin^2(x) = 1).$$

$$C = \cos\left(2 \times \frac{4\pi}{3}\right) = \cos\left(\frac{8\pi}{3}\right) = \cos\left(2\pi + \frac{2\pi}{3}\right) = \cos\left(\frac{2\pi}{3}\right) = \cos\left(\pi - \frac{\pi}{3}\right) = -\cos\left(\frac{\pi}{3}\right)$$

$$\text{donc } \boxed{C = -\frac{1}{2}}.$$

$$\begin{aligned} D &= \cos\left(\frac{\pi}{4}\right) + \cos\left(\pi - \frac{\pi}{4}\right) + \cos\left(\pi + \frac{\pi}{4}\right) + \cos\left(2\pi - \frac{\pi}{4}\right) \\ &= \cos\left(\frac{\pi}{4}\right) - \cos\left(\frac{\pi}{4}\right) - \cos\left(\frac{\pi}{4}\right) + \cos\left(-\frac{\pi}{4}\right) = -\cos\left(\frac{\pi}{4}\right) + \cos\left(\frac{\pi}{4}\right) \end{aligned}$$

$$\text{donc } \boxed{D = 0}.$$

$$\begin{aligned}
E &= \tan\left(\pi - \frac{\pi}{3}\right) + \tan\left(\pi - \frac{\pi}{4}\right) + \tan\left(\pi - \frac{\pi}{6}\right) + \tan\left(\pi + \frac{\pi}{6}\right) \\
&= \tan\left(-\frac{\pi}{3}\right) + \tan\left(-\frac{\pi}{4}\right) + \tan\left(-\frac{\pi}{6}\right) + \tan\left(\frac{\pi}{6}\right) \\
&= -\tan\left(\frac{\pi}{3}\right) - \tan\left(\frac{\pi}{4}\right) - \tan\left(\frac{\pi}{6}\right) + \tan\left(\frac{\pi}{6}\right) \\
\text{donc } &\boxed{E = -\sqrt{3} - 1}.
\end{aligned}$$

Exercice 3. Soit $x \in \mathbb{R}$. Simplifier :

$$\begin{aligned}
A &= \sin(\pi - x) + \cos\left(\frac{\pi}{2} + x\right) & B &= \sin(-x) + \cos(\pi + x) + \sin\left(\frac{\pi}{2} - x\right) \\
C &= \sin\left(\frac{\pi}{2} - x\right) + \sin\left(\frac{\pi}{2} + x\right) & D &= \cos(x - \pi) + \sin\left(-\frac{\pi}{2} - x\right)
\end{aligned}$$

Solution.

$$A = \sin(x) - \sin(x) \text{ donc } \boxed{A = 0}.$$

$$B = -\sin(x) - \cos(x) + \cos(x) \text{ donc } \boxed{B = -\sin(x)}.$$

$$C = \cos(x) + \cos(x) \text{ donc } \boxed{C = 2\cos(x)}$$

$$D = -\cos(x) - \cos(x) \text{ donc } \boxed{D = -2\cos(x)}.$$

Exercice 4. Sans chercher à les calculer, déterminer le signe de chacun des nombres suivants.

$$\begin{aligned}
A &= \cos\left(\frac{2\pi}{5}\right) & B &= \sin\left(\frac{8\pi}{5}\right) & C &= \sin\left(\frac{14\pi}{5}\right) \\
D &= \sin\left(\frac{7\pi}{5}\right) & E &= \tan\left(\frac{13\pi}{5}\right) & F &= \tan\left(-\frac{3\pi}{5}\right)
\end{aligned}$$

Solution.

$$\text{Comme } -\frac{\pi}{2} < \frac{2\pi}{5} < \frac{\pi}{2}, \boxed{A > 0}.$$

$$\text{Comme } \pi < \frac{8\pi}{5} < 2\pi, \boxed{B < 0}.$$

$$C = \sin\left(2\pi + \frac{4\pi}{5}\right) = \sin\left(\frac{4\pi}{5}\right) \text{ donc, comme } 0 < \frac{4\pi}{5} < \pi, \boxed{C > 0}.$$

$$\text{Comme } \pi < \frac{7\pi}{5} < 2\pi, \boxed{D < 0}.$$

$$E = \tan\left(\pi + \frac{3\pi}{5}\right) = \tan\left(\frac{3\pi}{5}\right). \text{ Or, comme } \frac{\pi}{2} < \frac{3\pi}{5} < \pi, \cos\left(\frac{3\pi}{5}\right) < 0 \text{ et } \sin\left(\frac{3\pi}{5}\right) > 0$$

donc $\boxed{E < 0}$.

$$F = -\tan\left(\frac{3\pi}{5}\right) = -E \text{ donc } \boxed{F > 0}.$$

Exercice 5. Soit $x \in \left]0; \frac{\pi}{2}\right[$. En utilisant les formules de duplication, simplifier :

$$A = \frac{1 - \cos(2x)}{\sin(2x)} \quad B = \frac{\cos(2x)}{\cos(x)} - \frac{\sin(2x)}{\sin(x)}$$

Solution.

$$A = \frac{1 - (1 - 2 \sin^2(x))}{2 \sin(x) \cos(x)} = \frac{2 \sin^2(x)}{2 \sin(x) \cos(x)} = \frac{\sin(x)}{\cos(x)} \text{ donc } \boxed{A = \tan(x)}.$$

$$B = \frac{\sin(x) \cos(2x) - \sin(2x) \cos(x)}{\cos(x) \sin(x)} = \frac{\sin(x - 2x)}{\sin(x) \cos(x)} = \frac{-\sin(x)}{\cos(x) \sin(x)} \text{ donc } \boxed{B = -\frac{1}{\cos(x)}}.$$

Exercice 6. Résoudre dans \mathbb{R} les équations suivantes d'inconnues x .

$$(E_1) : \cos(x) = \frac{1}{2} \quad (E_2) : \sin(x) = -\frac{\sqrt{3}}{2} \quad (E_3) : \sin(x) = \cos\left(\frac{2\pi}{3}\right) \quad \cos^2(x) = \frac{1}{2}.$$

Solution.

$$(E_1) \iff \cos(x) = \cos\left(\frac{\pi}{3}\right) \iff x = \frac{\pi}{3} [2\pi] \text{ ou } x = -\frac{\pi}{3} [2\pi]$$

Ainsi, l'ensemble des solutions de (E_1) est $\boxed{\left\{\frac{\pi}{3} + k2\pi \mid k \in \mathbb{Z}\right\} \cup \left\{-\frac{\pi}{3} + k2\pi \mid k \in \mathbb{Z}\right\}}$.

$$(E_2) \iff \sin(x) = \sin\left(-\frac{\pi}{3}\right) \iff x = -\frac{\pi}{3} [2\pi] \text{ ou } x = -\frac{2\pi}{3} [2\pi]$$

Ainsi, l'ensemble des solutions de (E_2) est $\boxed{\left\{-\frac{\pi}{3} + k2\pi \mid k \in \mathbb{Z}\right\} \cup \left\{-\frac{2\pi}{3} + k2\pi \mid k \in \mathbb{Z}\right\}}$.

$$(E_3) \iff \sin(x) = \sin\left(-\frac{\pi}{6}\right) \iff x = -\frac{\pi}{6} [2\pi] \text{ ou } x = -\frac{5\pi}{6} [2\pi]$$

Ainsi, l'ensemble des solutions de (E_3) est $\boxed{\left\{-\frac{\pi}{6} + k2\pi \mid k \in \mathbb{Z}\right\} \cup \left\{-\frac{5\pi}{6} + k2\pi \mid k \in \mathbb{Z}\right\}}$.

$$(E_4) \iff |\cos(x)| = \frac{1}{\sqrt{2}} \iff \cos(x) = \frac{\sqrt{2}}{2} \text{ ou } \cos(x) = -\frac{\sqrt{2}}{2}$$

$$\iff x = \frac{\pi}{4} [2\pi] \text{ ou } x = -\frac{\pi}{4} [2\pi] \text{ ou } x = \frac{3\pi}{4} [2\pi] \text{ ou } x = -\frac{3\pi}{4} [2\pi]$$

$$\iff x = \frac{\pi}{4} \left[\frac{\pi}{2}\right]$$

Ainsi, l'ensemble des solutions de (E_4) est $\boxed{\left\{\frac{\pi}{4} + k\frac{\pi}{2} \mid k \in \mathbb{Z}\right\}}$.