Corrigé des exercices donnés pour le vendredi 03 avril mars 2020

Exercice 79 p. 181

$I = \int_1^3 3x^2 - 2x \,\mathrm{d}x = \left[x^3 - x^2\right]_1^3 = 3^3 - 3^2 - (1^3 - 1^2) = 18$

$J = \int_{-3}^0 x^2 \,\mathrm{d}x = \left[\frac{x^3}{3}\right]_{-3}^0 = 0 - \frac{(-3)^3}{3} = 9$

$K = \int_1^3 \frac{2}{x} \,\mathrm{d}x = [2\ln(x)]_1^3 = 2\ln(3) - 2\ln(1) = 2\ln(3)$

Exercice 80 p. 181

$I = \int_{-1}^2 x^2 - 4x \,\mathrm{d}x = \left[\frac{x^3}{3} - 2x^2\right]_{-1}^2 = \frac{2^3}{3} - 2\times 2^2 - \left(\frac{(-1)^3}{3} - 2(-1)^3\right) = \frac{8}{3} - 8 + \frac{1}{3} + 2 = -3$

$J = \int_1^2 \frac{1}{x^2} \,\mathrm{d}x = \left[-\frac{1}{x}\right]_1^2 = -\frac{1}{2} - \left(-\frac{1}{1}\right) = \frac{1}{2}$

$K = \int_{-1}^1 \mathrm{e}^{-x} \,\mathrm{d}x = \left[-\mathrm{e}^{-x}\right]_{-1}^1 = -\mathrm{e}^{-1} - (-\mathrm{e}^{-(-1)}) = e - e^{-1}$

Exercice 81 p. 181

$I = \int_1^2 x^{-3} \,\mathrm{d}x = \int_1^2 \frac{1}{x^3} \,\mathrm{d}x \left[-\frac{1}{2x^2}\right]_1^2 = -\frac{1}{2\times 2^2} - \left(-\frac{1}{2\times 1^2}\right) = -\frac{1}{8} + \frac{1}{2} = \frac{3}{8}$

$J = \int_0^1 \frac{x}{x^2+3} \,\mathrm{d}x = \frac{1}{2} \int_0^1 \underbrace{\frac{2x}{x^2+3}}_{\frac{u'}{u}} \,\mathrm{d}x = \frac{1}{2}\left[\ln(x^2+3)\right]_0^1 = \frac{1}{2}(\ln(1^2+3) - \ln(0^2+3))$ donc

$J = \frac{1}{2}(\ln(4) - \ln(3)) = \frac{1}{2}\ln\left(\frac{4}{3}\right)$.

$K = \int_{-1}^2 x\mathrm{e}^{x^2} \,\mathrm{d}x = \frac{1}{2} \int_{-1}^2 \underbrace{2x\mathrm{e}^{x^2}}_{u'\mathrm{e}^u} \mathrm{d}x \frac{1}{2}\left[\mathrm{e}^{x^2}\right]_{-1}^2 = \frac{1}{2}(\mathrm{e}^{2^2} - \mathrm{e}^{(-1)^2}) = \frac{e^4 - e}{2}$

Exercice 82 p. 181

$I = \int_0^1 \mathrm{e}^{-x}\mathrm{e}^{-x} + 3 \,\mathrm{d}x = - \int_0^1 \underbrace{\frac{-\mathrm{e}^{-x}}{\mathrm{e}^{-x}+3}}_{\frac{u'}{u}} \,\mathrm{d}x = -\left[\ln(\mathrm{e}^{-x}+3)\right]_0^1 = -\left[\ln(\mathrm{e}^{-1}+3) - \ln(\mathrm{e}^0+3)\right]$

donc $I = -\ln(e^{-1}+3) + \ln(4) = \ln\left(\frac{4}{e^{-1}+3}\right)$

$J = \int_0^{\frac{\pi}{2}} \sin^2(x)\cos(x) \,\mathrm{d}x = \int_0^{\frac{\pi}{2}} \underbrace{\cos(x)\sin^2(x)}_{u'u^2} \,\mathrm{d}x = \left[\frac{1}{3}\sin^3(x)\right]_0^{\frac{\pi}{2}} = \frac{1}{3}\sin^3\left(\frac{\pi}{2}\right) - \frac{1}{3}\sin^3(0)$

donc $J = \frac{1}{3}$

$K = \int_0^1 \frac{1}{\sqrt{3x+1}} \,\mathrm{d}x = \frac{1}{3} \int_0^1 \underbrace{\frac{3}{\sqrt{3x+1}}}_{\frac{u'}{\sqrt{u}}} \,\mathrm{d}x = \frac{1}{3}\left[2\sqrt{3x+1}\right]_0^1 = \frac{1}{3}\left[2\sqrt{3\times 1+1} - 2\sqrt{3\times 0+1}\right]$

donc $K = \frac{2}{3}$